

Two Photon Exchange in Impact Parameter Space in the Relativistic Eikonal Approximation for Elastic $e - N^\uparrow$ Scattering

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Abstract

Using the relativistic Eikonal approximation, we study the one and two photon exchange amplitudes in elastic electron-nucleon scattering for the case of transversely polarized nucleons with unpolarized electrons beam. In our approach, we utilize the convolution theory of Fourier transforms and the transverse charge density in transverse momentum space to evaluate the one and two photon exchange Eikonal amplitudes. The results obtained for the 2γ amplitude in impact parameter space are compared to the corresponding 4-D case. We show that while the one and two photon cross sections are azimuthally symmetric, the interference term between them is azimuthally asymmetric, which is an indication of an azimuthal single spin asymmetry for proton and neutron which can be attributed to the fact that the nucleon charge density is transversely (azimuthally) distorted in the transverse plane for transversely polarized nucleons. In addition, the calculations of the interference term for proton and neutron show agreement in sign and magnitude of the existence data and calculations for transverse target single spin asymmetry.

I. Introduction

Two photon exchange has recently become an attractive subject due to its importance in the calculations of different quantities in elastic and deep inelastic scattering processes, for example single spin asymmetry is proportional to the interference of the one and two photon exchange amplitudes both in the elastic [1, 2] and deep inelastic [3, 4] cases. It is also recently noted that the two photon exchange calculations are important to increase the precision in related quantities, for example the need for precision to resolve the inconsistency in the measurement of the electromagnetic form factors ratio G_E/G_M using different methods of measurements [5–7], as shown in Figure.1.

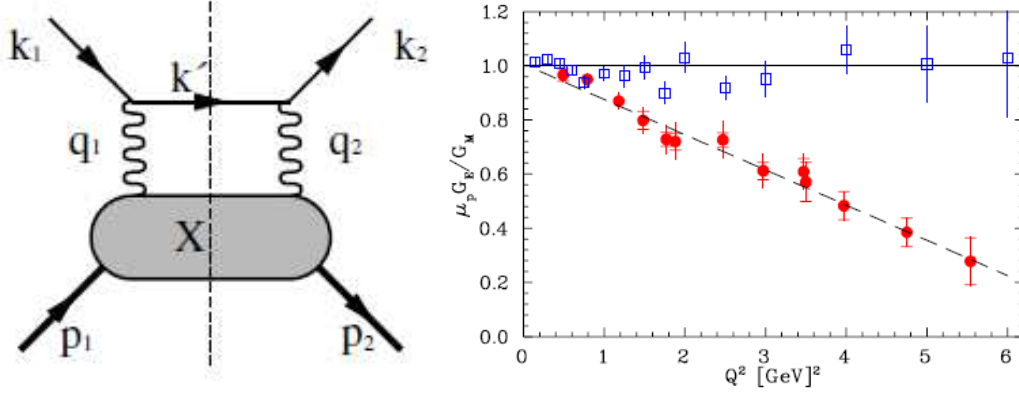


Figure 1: Left: two photon exchange box diagram. Right: Data of electromagnetic form factor ratio for proton, circles: polarization transfer, squares: cross section measurements (Rosenbluth technique).

The extension of the two photon exchange calculations of the unpolarized case to polarized cases is important especially when dealing with quantities that are directly related to the 2γ amplitude, such as single and double spin asymmetries. On the other hand, knowledge of the impact parameter dependence of the 2γ exchange process has the advantage of connecting it with different quantities in impact parameter space, which is currently of prime importance in studying the spin structure of the nucleon [8, 9] and in high energy elastic and deep inelastic scattering processes.

In this work, we study the one and two photon exchange amplitudes for transversely polarized target in elastic $e - N^\uparrow$ scattering in impact parameter space utilizing the relativistic Eikonal approximation. The obtained results are compared to the conventional four dimensional case of the two photon exchange process, where a notable similarity exist between the two cases; for example, as a result of our calculations, we noticed (for proton and neutron) that while the one and two photon elastic cross sections are azimuthally symmetric, the interference of the corresponding elastic one and two photon exchange amplitudes in impact parameter space is azimuthally asymmetric which is an indication of a non zero azimuthal single spin asymmetry for both proton and neutron. The calculations are done for GPD and Sachs parametrization of the nucleon's form factors for proton and neutron.

II. Nucleon Transverse Charge and Magnetization Densities

For a nucleon of mass M_N , transversely polarized (with respect to its momentum direction) in the x direction, the charge density is no longer axially symmetric and the distribution of partons in the transverse plane is given by

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp} \left[H^q(x, 0, -q_\perp^2) + i \frac{q_y}{2M_N} E^q(x, 0, -q_\perp^2) \right] = q(x, b_\perp) - \frac{1}{2M_N} \frac{\partial}{\partial b_y} \mathcal{E}^q(x, \mathbf{b}_\perp), \quad (1)$$

where

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} H^q(x, 0, -q_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp}. \quad (2)$$

$$\mathcal{E}^q(x, \mathbf{b}_\perp) = \int \frac{d^2 q_\perp}{(2\pi)^2} E^q(x, -q_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp}, \quad (3)$$

here $\mathbf{b}_\perp = b_\perp \cos(\phi_{b_\perp}) \hat{e}_x + b_\perp \sin(\phi_{b_\perp}) \hat{e}_y$ is the transverse (impact parameter) vector, $\mathbf{q}_\perp = q_\perp \cos(\phi_{q_\perp}) \hat{e}_x + q_\perp \sin(\phi_{q_\perp}) \hat{e}_y$ is the transverse momentum transfer and the nucleon spin is transverse to the incident beam direction with spin vector $\mathbf{S} = \cos(\phi_s) \hat{e}_x + \sin(\phi_s) \hat{e}_y$. Now using

$$F_1(t) = \sum_q e_q \int_{-1}^1 dx H_q(x, \xi, t), \quad F_2(t) = \sum_q e_q \int_{-1}^1 dx E_q(x, \xi, t), \quad (4)$$

and

$$\rho_1(|\mathbf{b}_\perp|) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} F_1(q_\perp^2), \quad \rho_2(|\mathbf{b}_\perp|) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} F_2(q_\perp^2). \quad (5)$$

we get from Eq.(1)

$$\rho(\mathbf{b}_\perp) = \rho_1(|\mathbf{b}_\perp|) - \frac{1}{2M_N} \frac{\partial}{\partial y} \rho_2(|\mathbf{b}_\perp|), \quad (6)$$

Evaluating the Fourier transform in Eq.(1) one obtains the transverse charge density in impact parameter space [10]

$$\rho_T(\mathbf{b}_\perp) = \rho(b_\perp) - \sin(\phi_b - \phi_s) \int_0^\infty \frac{dq_\perp}{2\pi} \frac{q_\perp^2}{2M_N} j_1(b_\perp q_\perp) F_2(q_\perp^2). \quad (7)$$

The charge density in transverse momentum space is obtained by taking the Fourier transform of Eq.(1) (or Eq.(7))

$$\tilde{\rho}_T(\mathbf{x}, \mathbf{q}_\perp) = H^q(x, -q_\perp^2) + i \frac{q_y}{2M_N} E^q(x, -q_\perp^2), \quad (8)$$

which implies

$$\tilde{\rho}_T(\mathbf{q}_\perp) = F_1(q_\perp^2) + i \frac{q_y}{2M_N} F_2(q_\perp^2) \quad (9)$$

III. One and Two Photon Exchange Amplitudes in the Relativistic Eikonal Approximation

For unpolarized beam of electrons elastically scattered from a transversely polarized nucleon target, the scattering amplitude in the relativistic Eikonal approximation is given by [11]

$$f^\uparrow(\mathbf{q}_\perp) = -2is \int d^2 b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[e^{i\chi^\uparrow(\mathbf{b}_\perp)} - 1 \right], \quad (10)$$

where s is the center of mass energy and $\chi^\uparrow(\mathbf{b}_\perp)$ is the Coulomb/Eikonal phase associated with a nucleon target of spin transverse to the incident beam direction. The momentum transfer is assumed to be purely transverse, i.e. $\vec{q}_\perp = \vec{q} - q_z \hat{z}$. The Coulomb/Eikonal phase is given by

$$\chi^\uparrow(\mathbf{b}_\perp) = \frac{-4\pi\alpha}{2s} \int_{-\infty}^{\infty} dz A^{(0)\uparrow}(\mathbf{b}_\perp, z) = \frac{-4\pi\alpha}{2s} A^{(0)\uparrow}(\mathbf{b}_\perp), \quad (11)$$

here $A^{(0)\uparrow}$ is the electromagnetic transverse potential produced by a transversely polarized nucleon of spin up, and $\alpha = 1/137$ is the electromagnetic coupling constant. Therefore, the Eikonal scattering amplitude becomes after expanding Eq.(10)

$$\begin{aligned} f^\uparrow(\mathbf{q}_\perp) = & -2is \int d^2 b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[e^{\frac{-i4\pi\alpha}{2s} A^{(0)\uparrow}(\mathbf{b}_\perp)} - 1 \right] = \\ & -4\pi\alpha \int d^2 b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} A^{(0)\uparrow}(\mathbf{b}_\perp) + \frac{i8\pi\alpha^2}{s} \int d^2 b_\perp e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \left[A^{(0)\uparrow}(\mathbf{b}_\perp) \right]^2 + \dots \end{aligned} \quad (12)$$

The above expansion is represented by the following diagram

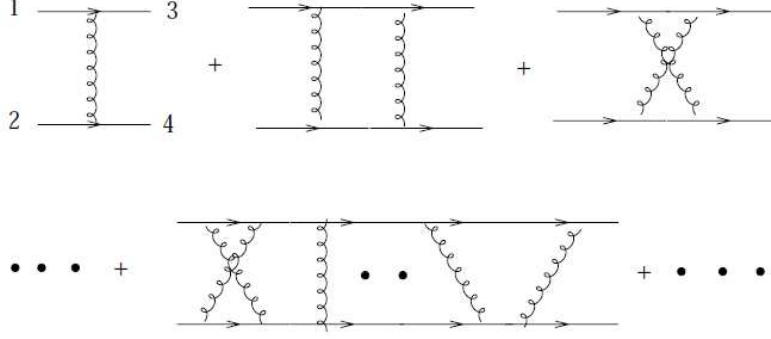


Figure 2: The sum of the diagrams resulting from Eikonal amplitude expansion . Figure from Ref [12].

Thus, the first term in the Eikonal approximation (Born approximation) reads

$$f_{1\gamma}^{\uparrow}(\mathbf{q}_{\perp}) = -4\pi\alpha \int d^2b_{\perp} e^{-i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}} A^{(0)\uparrow}(\mathbf{b}_{\perp}), \quad (13)$$

utilizing the Fourier transform of $\nabla^2 A^{(0)\uparrow}(\mathbf{b}_{\perp}) = -\rho(\mathbf{b}_{\perp})$ we get

$$f_{1\gamma}^{\uparrow}(\mathbf{q}_{\perp}) = -4\pi\alpha \tilde{A}^{(0)\uparrow}(\mathbf{q}_{\perp}) = -4\pi\alpha \frac{\tilde{\rho}(\mathbf{q}_{\perp})}{q_{\perp}^2}, \quad (14)$$

using Eq.(9), the one photon exchange amplitude becomes

$$f_{1\gamma}^{\uparrow}(\mathbf{q}_{\perp}) = A \left[F_1(q_{\perp}^2) + \frac{iq_{\perp} \sin(\phi_{q_{\perp}} - \phi_s)}{2M_N} F_2(q_{\perp}^2) \right], \quad (15)$$

where $A = -\frac{4\pi\alpha}{q_{\perp}^2}$ and M_N is the nucleon mass. Clearly, the (1γ) exchange amplitude depends on the azimuthal angle however, the corresponding cross section is azimuthally symmetric.

Next we consider the two photon exchange amplitude, which from Eq.(12) reads

$$f_{2\gamma}^{\uparrow}(\mathbf{q}_{\perp}) = i\frac{8\pi\alpha^2}{s} \int d^2b_{\perp} e^{-i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}} \left(A^{(0)\uparrow}(\mathbf{b}_{\perp}) \right)^2, \quad (16)$$

from the above formula for $f_{2\gamma}^{\uparrow}(\mathbf{q}_{\perp})$, we also see that while the amplitude depends on the azimuthal angle, the corresponding cross section gives no asymmetry (similar to $f_{1\gamma}^{\uparrow}(\mathbf{q}_{\perp})$) in the scattering cross section. In order to evaluate $f_{2\gamma}^{\uparrow}(\mathbf{q}_{\perp})$, we rewrite it in the form

$$f_{2\gamma}^{\uparrow}(\mathbf{q}_{\perp}) = iB \int \frac{d^2b_{\perp}}{(2\pi)^2} e^{-i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}} A^{(0)\uparrow}(\mathbf{b}_{\perp}) A^{(0)\uparrow}(\mathbf{b}_{\perp}), \quad (17)$$

where $B = \frac{8\pi\alpha^2}{s}$. The above integral represents the Fourier transform of the product of two functions and can be rewritten using the convolution theorem of Fourier transforms as

$$\begin{aligned} f_{2\gamma}^{\uparrow}(\mathbf{q}_{\perp}) &= iB \int \frac{d^2q'_{\perp}}{(2\pi)^2} \tilde{A}^{(0)\uparrow}(\mathbf{q}'_{\perp}) \tilde{A}^{(0)\uparrow}(\mathbf{q}_{\perp} - \mathbf{q}'_{\perp}) \\ &= iB \int \frac{d^2q'_{\perp}}{(2\pi)^2} \frac{\tilde{\rho}(\mathbf{q}'_{\perp})}{\mathbf{q}'_{\perp}{}^2} \frac{\tilde{\rho}(\mathbf{q}_{\perp} - \mathbf{q}'_{\perp})}{|\mathbf{q}_{\perp} - \mathbf{q}'_{\perp}|^2}, \end{aligned} \quad (18)$$

using Eq.(9), $f_{2\gamma}^\uparrow(\mathbf{q}_\perp)$ becomes

$$f_{2\gamma}^\uparrow(\mathbf{q}_\perp) = iB \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{1}{|\mathbf{q}'_\perp|^2} \left[F_1(|\mathbf{q}'_\perp|^2) + \frac{iq'_y}{2M_N} F_2(|\mathbf{q}'_\perp|^2) \right] \frac{1}{|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} \left[F_1(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2) + \frac{i(q_y - q'_y)}{2M_N} F_2(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2) \right], \quad (19)$$

clearly the two photon exchange contribution appears due to the convolution between different combinations of Dirac and Pauli form factors, in contrast to the conventional 4-D case where extra form factors are introduced to define the two photon exchange amplitude. Now from the above equation, we see that $f_{2\gamma}^\uparrow(\mathbf{q}_\perp)$ contains the following integrals

$$I_1 = \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{1}{|\mathbf{q}'_\perp|^2 |\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} F_1(|\mathbf{q}'_\perp|^2) F_1(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2), \quad (20)$$

$$I_2 = \frac{1}{2M_N} \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{(q_y - q'_y)}{|\mathbf{q}'_\perp|^2 |\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} F_1(|\mathbf{q}'_\perp|^2) F_2(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2), \quad (21)$$

$$I_3 = \frac{1}{2M_N} \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{q'_y}{|\mathbf{q}'_\perp|^2 |\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} F_1(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2) F_2(|\mathbf{q}'_\perp|^2), \quad (22)$$

$$I_4 = \frac{1}{4M_N^2} \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{q'_y(q_y - q'_y)}{|\mathbf{q}'_\perp|^2 |\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} F_2(|\mathbf{q}'_\perp|^2) F_2(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2), \quad (23)$$

where

$$f_{2\gamma}^\uparrow(\mathbf{q}_\perp) = iB [I_1 + i I_2 + i I_3 - I_4]. \quad (24)$$

Performing the change of variables $\mathbf{q}''_\perp = \mathbf{q}_\perp - \mathbf{q}'_\perp$ in I_2 , one gets

$$I_2 = \frac{1}{2M_N} \int \frac{d^2 q''_\perp}{(2\pi)^2} \frac{q''_y}{|\mathbf{q}''_\perp|^2 |\mathbf{q}_\perp|^2} F_1(|\mathbf{q}''_\perp|^2) F_2(|\mathbf{q}_\perp|^2), \quad (25)$$

therefore, $I_2 = I_3$, and $f_{2\gamma}^\uparrow$ becomes (noting that $q_y = q_\perp \sin(\phi_{q_\perp} - \phi_s)$)

$$\begin{aligned} f_{2\gamma}^\uparrow(\mathbf{q}_\perp) &= iB [I_1 + 2i I_3 - I_4] \\ &= iB [I_1 + I_{41} - q_\perp \sin(\phi_{q_\perp} - \phi_s) I_{42} + 2i I_3] \\ &= iB [I_1 + I_{41} - q_\perp \sin(\phi_{q_\perp} - \phi_s) I_{42}] - 2B I_3, \end{aligned} \quad (26)$$

where

$$\begin{aligned} I_{41} &= \frac{1}{4M_N^2} \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{q_y'^2}{|\mathbf{q}'_\perp|^2 |\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} F_2(|\mathbf{q}'_\perp|^2) F_2(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2), \\ I_{42} &= \frac{1}{4M_N^2} \int \frac{d^2 q'_\perp}{(2\pi)^2} \frac{q'_y}{|\mathbf{q}'_\perp|^2 |\mathbf{q}_\perp - \mathbf{q}'_\perp|^2} F_2(|\mathbf{q}'_\perp|^2) F_2(|\mathbf{q}_\perp - \mathbf{q}'_\perp|^2). \end{aligned} \quad (27)$$

Note that I_2 , I_{41} and I_{42} are free from IR divergences (considering polar coordinates and the symmetry between the poles at $q'_\perp = 0$ and $q'_\perp = q_\perp$), while I_1 is IR divergent. To extract the IR divergence in I_1 we add a photon mass to the divergent part and use dimensional regularization, as illustrated in the next section.

IV. Isolating the IR Divergence in the Two Photon Exchange Amplitude Using Dimensional Regularization

The integrals I_{41} and I_{42} in Eq.(27) are free from IR divergences and can be evaluated numerically, while I_1 in Eq.(20) contains IR divergence that one needs to deal with. In the rest of this section (and appendix A), we use dipole parametrization for $F_1(q_\perp^2)$ and $F_2(q_\perp^2)$ to show that the behavior of the IR divergence in impact parameter space is similar to that in conventional 4-D space [1, 2, 13, 14], where it was found that the IR divergence cancels with the Bremsstrahlung contribution to the two photon amplitude (The details of the Bremsstrahlung calculations and the proof that it cancels with the IR divergence in the 2γ amplitude are found in the above references). Thus it is tempting to start by studying the behavior of the IR divergence of the 2γ amplitude in impact parameter space and compare the result with the 4-D case, to do this we first use the following dipole parametrization for the form factors [15], (Sachs form factors can be used as well and the same analysis used here can be employed)

$$F_1(q_\perp^2) = \frac{F_1(0)}{(1 + q_\perp^2/M_d^2)^2}, \quad F_2(q_\perp^2) = \frac{F_2(0)}{(1 + q_\perp^2/M_d^2)^2}, \quad (28)$$

where $F_1(0) = 1$, $F_2(0) = \kappa_p = 1.79$, $M_d^2 = 0.71 \text{ GeV}^2$. Introducing a virtual photon mass m' to regularize the IR divergence at $q'_\perp = 0$ and $q'_\perp = q_\perp$ and using dimensional regularization with $D = 2 - \epsilon$, I_1 becomes

$$I_1 = \int \frac{d^D q'_\perp}{(2\pi)^D} \frac{1}{(|\mathbf{q}'_\perp|^2 + m'^2) (|\mathbf{q}'_\perp - \mathbf{q}_\perp|^2 + m'^2)} \times \frac{M_d^8}{(|\mathbf{q}'_\perp|^2 + M_d^2)^2 (|\mathbf{q}'_\perp - \mathbf{q}_\perp|^2 + M_d^2)^2}. \quad (29)$$

Using partial fractions, the above integral can be decomposed into (9) integrals (the same result can be obtained using the package FeynCalc [16]) which can be evaluated using dimensional regularization (appendix A contains detailed calculations), where all the IR divergences cancel except a logarithmic term that appears due to the following integral (I_{15} in appendix A)

$$I_{1IR} = \frac{M_d^8}{(m'^2 - M_d^2)^4} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(m'^2 + \mathbf{q}'_\perp^2) [m'^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]}, \quad (30)$$

using Feynman parametrization we get

$$\begin{aligned} I_{1IR} &= \frac{M_d^8 \Gamma(1)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(1) \Gamma(1)} \int_0^1 \frac{1}{[m'^2 + q_\perp^2 x(1-x)]^{1+\frac{\epsilon}{2}}} dx \\ &= \frac{2M_d^8}{2\pi(m'^2 - M_d^2)^3 q_\perp \sqrt{4m'^2 + q_\perp^2}} \ln \left(\frac{q_\perp (\sqrt{4m'^2 + q_\perp^2} + q_\perp) + 2m'^2}{2m'^2} \right) \\ &\approx \frac{-M_d^2}{\pi q_\perp^2} \ln \left(\frac{q_\perp^2}{m'^2} \right). \end{aligned} \quad (31)$$

Similar decomposition can be done for I_{41} and I_{42} in Eq.(27) from which one can show, using dimensional regularization, that all the IR divergences cancel each other, however, I_{41} and I_{42} can be evaluated numerically using polar coordinates. On the other hand, the logarithmic result of the IR divergence of the 2γ amplitude in impact parameter space is similar to that in the $4 - D$ case [2, 13]. In appendix A, we show using dimensional regularization the full evaluation of I_1 using the dipole form factors where it is noted that all the IR divergences cancel except a logarithmic term that appears in the following integral

(see appendix A for the details)

$$\begin{aligned}
I_{15} &= \frac{1}{(m'^2 - M_d^2)^4} \int \frac{d^D \mathbf{q}'_{\perp}}{(2\pi)^D} \frac{1}{(m'^2 + \mathbf{q}'_{\perp}^2) [m'^2 + (\mathbf{q}'_{\perp} - \mathbf{q}_{\perp})^2]} \\
&= \frac{\Gamma(1)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(1) \Gamma(1)} \int_0^1 \frac{1}{[m'^2 + q_{\perp}^2 x(1-x)]^{1+\frac{\epsilon}{2}}} dx \\
&= \frac{1}{2\pi(m'^2 - M_d^2)^3} \frac{2 \ln \left(\frac{q(\sqrt{4m'^2 + q^2} + q) + 2m'^2}{2m'^2} \right)}{q \sqrt{4m'^2 + q^2}},
\end{aligned} \tag{32}$$

this result reduces to the result obtained in Eq.(31) using the leading order of the integral. Again we emphasize that the full analysis shown in appendix A can be applied to Sachs form factors, however the number of integrals resulting from using dimensional regularization will be large. Therefore it is useful to try to evaluate the 2γ exchange integrals numerically, which also allow us to use other parametrizations of the form factors, such that generalized parton distributions parametrization, *GPDs*. The only *IR* divergence that we need to take into account is that appears in I_1 since in the other integrals the divergences canceled when using polar coordinates and the symmetry of the propagators. In the following section we show a possible way deal with such divergence.

V. Evaluation of the Two Photon Exchange Amplitude for Arbitrary Parametrization of the Form Factors

The integrals in Eqs.(26) can be evaluated numerically for any parametrization of the form factors. For I_1 , the *IR* divergence can be extracted by expanding the propagator $\frac{1}{|\mathbf{q}'_{\perp} - \mathbf{q}_{\perp}|^2}$ in Eq.(20) (see appendix B for the details)

$$I_1 = \frac{1}{q_{\perp}^2} \int \frac{d\phi' dq'_{\perp}}{(2\pi)^2} \left[\frac{2}{q_{\perp}} \cos(\phi - \phi') + \frac{q'_{\perp}}{q_{\perp}^2} (\cos^2(\phi - \phi') + 3 \cos(\phi - \phi') - 1) + \dots \right] \times F_1(|\mathbf{q}'_{\perp}|^2) F_1(|\mathbf{q}'_{\perp} - \mathbf{q}_{\perp}|^2) \tag{33}$$

The above expansion follows from the addition theorem of spherical harmonics for $q'_{\perp} < q_{\perp}$ for which the charge density is represented by the transverse charge density of the nucleon. In the numerical calculations, the size of the nucleon in the transverse (or impact parameter) plane was taken from references [9,10] and we used the leading order of the finite part of the above expansion after subtracting the *IR* divergence that leads to the logarithmic divergence which at the end cancels with the Bremsstrahlung contribution as shown in the previous section.

As shown in section III, the one and two photon amplitudes are azimuthally symmetric and therefore we do not expect any asymmetry from the corresponding cross sections. However, the interference term of the 1γ and 2γ amplitudes is azimuthally asymmetric, and *SSA* appears due to this term. Figure 3 shows the numerical calculations of the azimuthal distribution of the interference between the 1γ and 2γ amplitudes normalized to the 1γ (Born) cross section for proton and neutron at two different momentum transfers. In Figure 4 the ratio of the total cross section to the Born term is shown which is consistent with the 4-D case [7], the 2γ contribution (for proton and neutron) to the elastic scattering with nucleon intermediate state is shown in Figure 5. Since single spin asymmetry is proportional to the interference of the 1γ and 2γ amplitudes [7,17], the amplitudes in Figure 3 are a measure to this asymmetry. On the other hand, these plots show opposite signs for proton and neutron, which is an indication of the sign of azimuthal *SSA* for unpolarized electrons scattered from transversely polarized nucleon and is consistent with the proton results [1,7] and neutron results (using transversely polarized ^3He) in recent Jlap measurements for neutron [18]. A more detailed study of the transverse target azimuthal *SSA* using the transverse

electromagnetic potential associated with a transversely polarized nucleons is currently under preparation. The parametrization of the Sachs G_E and G_M form factors (appendix C_1) were taken from Ref [19] and the GPD parametrizations (appendix C_2) used were taken from Ref [20].

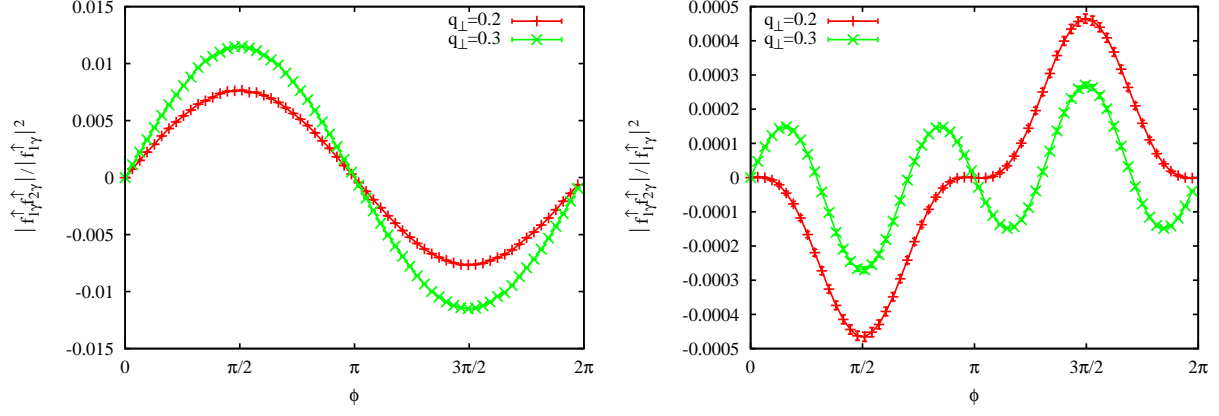


Figure 3: Interference of 1γ and 2γ amplitudes normalized to Born cross section for proton (left) and neutron (right)

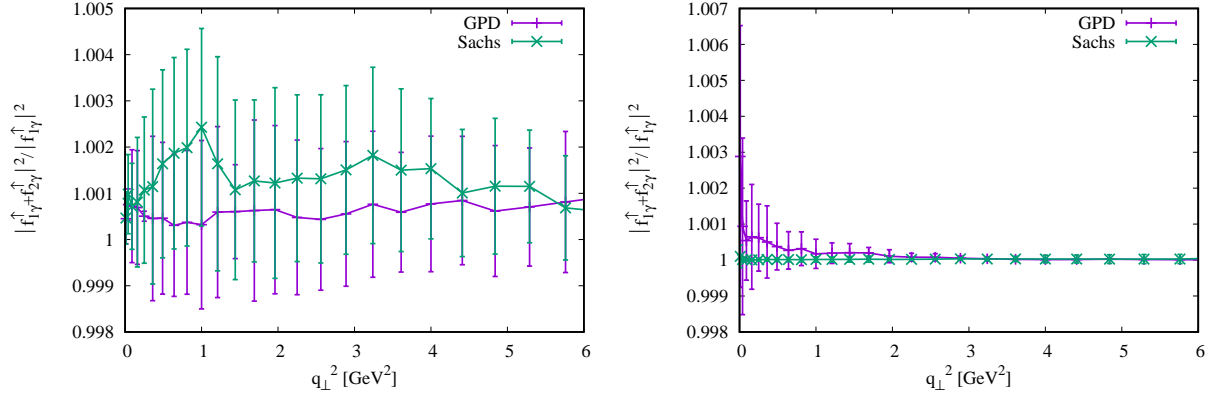


Figure 4: Ratio of the Born + 2γ contribution to elastic cross section for proton (left) and neutron (right) relative to Born term, using GPD and Sachs parametrization of the form factors

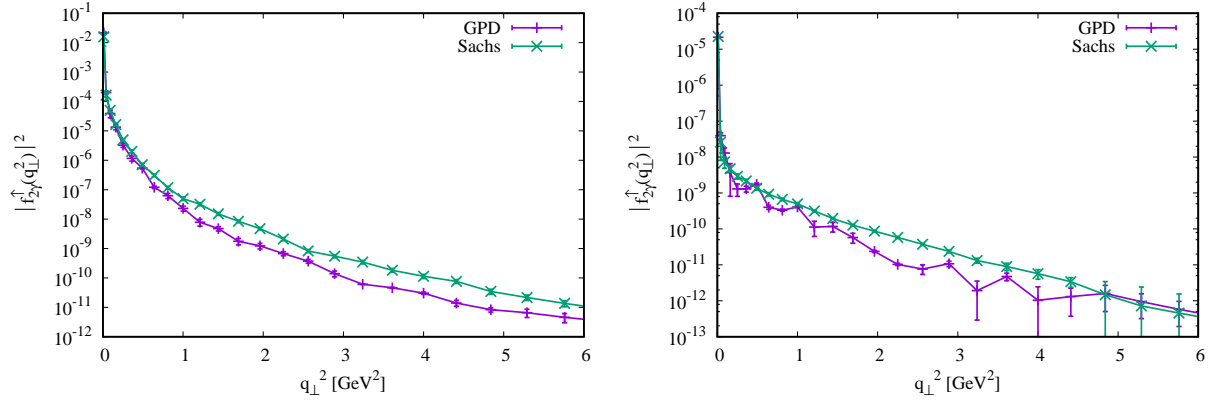


Figure 5: 2γ contribution to elastic cross section (in GeV^{-2}) for proton (left) and neutron (right) using GPD and Sachs parametrization of the form factors

VI. Conclusion

In this paper, we calculate the one and two photon exchange Eikonal amplitudes in impact parameter space for elastic $e - N^\uparrow$ scattering. The results show that the two photon exchange amplitude appears due to the convolution between different combinations of Dirac and Pauli form factors. On the other hand, while the amplitudes associated with the one and two photon exchanges are azimuthally asymmetric in the transverse plane, the corresponding cross sections are not. However, the interference term between the 1γ and 2γ amplitudes is azimuthally asymmetric, which is consistent with elastic and deep inelastic scattering for the 4-D case, and an indication of the existence of azimuthal single spin asymmetry in elastic scattering for both proton and neutron which can be attributed to the fact that the nucleon charge density is transversely (azimuthally) distorted in the transverse plane for transversely polarized nucleons. It is also noted that the two photon amplitude in impact parameter space contains an IR divergence that has the same logarithmic behavior as in the 4-D case, which cancels with the Bremsstrahlung contribution to the cross section due to real photon emission from the target and the beam.

As a future work and a direct consequence of the results of this paper, is the calculation of the target azimuthal single spin asymmetry in elastic scattering for transversely polarized protons and neutrons utilizing a recent calculation of the corresponding transverse electromagnetic potential. Another possible consequence of this work is to extend the calculations of the 2γ exchange amplitude in transverse plane to different processes in deep inelastic scattering, which should lead to the calculation of SSA for such processes.

Appendix A Full Dimensional Regularization Evaluation of I_1 for the Dipole Parametrization in Eq.(29)

We start by decomposing the integrand of I_1 given in Eq.(29) using partial fractions, this will allow us to easily use dimensional regularization to evaluate I_1 , noting that

$$\frac{1}{(x^2 + a)(x^2 + b)^2} = \frac{1}{(a - b)(b + x^2)^2} + \frac{1}{(a - b)^2(a + x^2)} - \frac{1}{(a - b)^2(b + x^2)}, \quad (34)$$

we have

$$\frac{1}{(\mathbf{q}_\perp^2 + m'^2)(\mathbf{q}_\perp^2 + M_d^2)^2} = \frac{1}{(m'^2 - M_d^2)(M_d^2 + \mathbf{q}_\perp^2)^2} + \frac{1}{(m'^2 - M_d^2)^2(m'^2 + \mathbf{q}_\perp^2)} - \frac{1}{(m'^2 - M_d^2)^2(M_d^2 + \mathbf{q}_\perp^2)}, \quad (35)$$

and

$$\begin{aligned} & \frac{1}{[(\mathbf{q}'_\perp - \mathbf{q}_\perp)^2 + m'^2] [(\mathbf{q}'_\perp - \mathbf{q}_\perp)^2 + M_d^2]^2} = \\ & \frac{1}{(m'^2 - M_d^2)(M_d^2 + [(\mathbf{q}'_\perp - \mathbf{q}_\perp)^2 + M_d^2]^2)} + \frac{1}{(m'^2 - M_d^2)^2(m'^2 + [(\mathbf{q}'_\perp - \mathbf{q}_\perp)^2 + M_d^2]^2)} \\ & - \frac{1}{(m'^2 - M_d^2)^2(M_d^2 + [(\mathbf{q}'_\perp - \mathbf{q}_\perp)^2 + M_d^2]^2)} \end{aligned} \quad (36)$$

Therefore I_1 , (Eq.(29)) consists of the following integrals

$$I_{11} = \frac{1}{(m'^2 - M_d^2)^2} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(M_d^2 + \mathbf{q}'_\perp^2)^2 [M_d^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]^2}, \quad (37)$$

$$I_{12} = \frac{1}{(m'^2 - M_d^2)^3} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(M_d^2 + \mathbf{q}'_\perp^2)^2 [m'^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]}, \quad (38)$$

$$I_{13} = -\frac{1}{(m'^2 - M_d^2)^3} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(M_d^2 + \mathbf{q}'_\perp^2)^2 [M_d^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]^2}, \quad (39)$$

$$I_{14} = \frac{1}{(m'^2 - M_d^2)^3} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(m'^2 + \mathbf{q}'_\perp^2) [M_d^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]^2}, \quad (40)$$

$$I_{15} = \frac{1}{(m'^2 - M_d^2)^4} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(m'^2 + \mathbf{q}'_\perp^2) [m'^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]}, \quad (41)$$

$$I_{16} = -\frac{1}{(m'^2 - M_d^2)^4} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(m'^2 + \mathbf{q}'_\perp^2) [M_d^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]}, \quad (42)$$

$$I_{17} = -\frac{1}{(m'^2 - M_d^2)^3} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(M_d^2 + \mathbf{q}'_\perp^2) [M_d^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]^2}, \quad (43)$$

$$I_{18} = -\frac{1}{(m'^2 - M_d^2)^4} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(M_d^2 + \mathbf{q}'_\perp^2) [m'^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]}, \quad (44)$$

$$I_{19} = \frac{1}{(m'^2 - M_d^2)^4} \int \frac{d^D \mathbf{q}'_\perp}{(2\pi)^D} \frac{1}{(M_d^2 + \mathbf{q}'_\perp^2) [M_d^2 + (\mathbf{q}'_\perp - \mathbf{q}_\perp)^2]}. \quad (45)$$

Using Feynman parametrization, and evaluating the momentum integrals for $D = 2 - \epsilon$, one gets

$$\begin{aligned}
I_{11} &= \frac{\Gamma(3)}{(m'^2 - M_d^2)^2 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(2) \Gamma(2)} \int_0^1 \frac{(1-x)x}{[M_d^2 + q_\perp^2 x(1-x)]^{3+\frac{\epsilon}{2}}} dx, \\
I_{12} &= \frac{\Gamma(2)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(2) \Gamma(1)} \int_0^1 \frac{(1-x)}{[M_d^2 + q_\perp^2 x(1-x) + (m'^2 - M_d^2)x]^{2+\frac{\epsilon}{2}}} dx, \\
I_{13} &= -\frac{\Gamma(2)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(2) \Gamma(1)} \int_0^1 \frac{(1-x)}{[M_d^2 + q_\perp^2 x(1-x)]^{2+\frac{\epsilon}{2}}} dx, \\
I_{14} &= \frac{\Gamma(2)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(2) \Gamma(1)} \int_0^1 \frac{(1-x)}{[m'^2 + q_\perp^2 x(1-x) + (M_d^2 - m'^2)x]^{2+\frac{\epsilon}{2}}} dx, \\
I_{15} &= \frac{\Gamma(1)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(1) \Gamma(1)} \int_0^1 \frac{1}{[m'^2 + q_\perp^2 x(1-x)]^{1+\frac{\epsilon}{2}}} dx, \\
I_{16} &= \frac{-\Gamma(1)}{(m'^2 - M_d^2)^4 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(2) \Gamma(1)} \int_0^1 \frac{1}{[m'^2 + q_\perp^2 x(1-x) + (M_d^2 - m'^2)x]^{1+\frac{\epsilon}{2}}} dx, \\
I_{17} &= -\frac{\Gamma(2)}{(m'^2 - M_d^2)^3 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(1) \Gamma(2)} \int_0^1 \frac{x}{[M_d^2 + q_\perp^2 x(1-x)]^{2+\frac{\epsilon}{2}}} dx, \\
I_{18} &= \frac{-\Gamma(1)}{(m'^2 - M_d^2)^4 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(1) \Gamma(1)} \int_0^1 \frac{1}{[M^2 + q_\perp^2 x(1-x) + (m'^2 - M_d^2)x]^{1+\frac{\epsilon}{2}}} dx, \\
I_{19} &= \frac{\Gamma(1)}{(m'^2 - M_d^2)^4 (2\pi)^{1-\frac{\epsilon}{2}} \Gamma(1) \Gamma(1)} \int_0^1 \frac{1}{[M^2 + q_\perp^2 x(1-x)]^{1+\frac{\epsilon}{2}}} dx.
\end{aligned} \tag{46}$$

Using Mathematica, one obtains for the integrals over x (these are the above integrals without the mass factor in the left hand side of each integrals)

$$\begin{aligned}
I_{11x} &= \left(q (q^2 - 2M^2) \sqrt{4M^2 + q^2} + 2M^2 (M^2 + q^2) \ln \left(\frac{\sqrt{4M^2 + q^2} + q}{\sqrt{4M^2 + q^2} - q} \right) - \right. \\
&\quad \left. 2M^2 (M^2 + q^2) \ln \left(\frac{\sqrt{4M^2 + q^2} - q}{\sqrt{4M^2 + q^2} + q} \right) \right) / M^2 q^3 (4M^2 + q^2)^{5/2}, \tag{47}
\end{aligned}$$

$$\begin{aligned}
I_{12x} &= \left[(m'^2 - M^2 + q^2) \sqrt{m'^4 - 2m'^2 (M^2 - q^2) + (M^2 + q^2)^2} - \right. \\
&\quad M^2 (-m'^2 + M^2 + q^2) \times \\
&\quad \ln \left(\frac{-m'^2 + \sqrt{m'^4 - 2m'^2 (M^2 - q^2) + (M^2 + q^2)^2} + M^2 - q^2}{m'^2 + \sqrt{m'^4 - 2m'^2 (M^2 - q^2) + (M^2 + q^2)^2} - M^2 + q^2} \right) - \\
&\quad M^2 (-m'^2 + M^2 + q^2) \times \\
&\quad \left. \ln \left(\frac{m'^2 + \sqrt{m'^4 - 2m'^2 (M^2 - q^2) + (M^2 + q^2)^2} - M^2 - q^2}{-m'^2 + \sqrt{m'^4 - 2m'^2 (M^2 - q^2) + (M^2 + q^2)^2} + M^2 + q^2} \right) \right] \\
&\quad / M^2 (m'^4 - 2m'^2 (M^2 - q^2) + (M^2 + q^2)^2)^{3/2}
\end{aligned} \tag{48}$$

$$I_{13x} = \frac{4 \tanh^{-1} \left(\frac{q}{\sqrt{4M^2 + q^2}} \right)}{q (4M^2 + q^2)^{3/2}} + \frac{1}{4M^4 + M^2 q^2}, \tag{49}$$

$$\begin{aligned}
I_{14x} = & \left[(-m'^2 + M^2 + q^2) \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} \right. \\
& - m'^2 (m'^2 - M^2 + q^2) \times \\
& \ln \left(\frac{-m'^2 + \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} + M^2 - q^2}{m'^2 + \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} - M^2 + q^2} \right) \\
& - m'^2 (m'^2 - M^2 + q^2) \times \\
& \left. \ln \left(\frac{m'^2 + \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} - M^2 - q^2}{-m'^2 + \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} + M^2 + q^2} \right) \right] \\
& / m'^2 \left(m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2 \right)^{3/2},
\end{aligned} \tag{50}$$

$$I_{15x} = \frac{2 \ln \left(\frac{q(\sqrt{4m'^2 + q^2} + q) + 2m'^2}{2m'^2} \right)}{q \sqrt{4m'^2 + q^2}}, \tag{51}$$

$$I_{16x} = \frac{\ln \left(\frac{m'^2 + \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} + M^2 + q^2}{m'^2 - \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} + M^2 + q^2} \right)}{\sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2}}, \tag{52}$$

$$I_{17x} = \frac{4 \tanh^{-1} \left(\frac{q}{\sqrt{4M^2 + q^2}} \right)}{q (4M^2 + q^2)^{3/2}} + \frac{1}{4M^4 + M^2 q^2}, \tag{53}$$

$$I_{18x} = \frac{\ln \left(\frac{m'^2 + \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} + M^2 + q^2}{m'^2 - \sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2} + M^2 + q^2} \right)}{\sqrt{m'^4 - 2m'^2(M^2 - q^2) + (M^2 + q^2)^2}}, \tag{54}$$

$$I_{19x} = \frac{2 \ln \left(\frac{q(\sqrt{4M^2 + q^2} + q) + 2M^2}{2M^2} \right)}{q \sqrt{4M^2 + q^2}}. \tag{55}$$

Appendix B Expansion of $\frac{1}{|q'_\perp - q_\perp|}$ in Impact Parameter Space

The impact parameter space expansion of $\frac{1}{|q'_\perp - q_\perp|}$ can be obtained using

$$\frac{1}{|\mathbf{q}_\perp - \mathbf{q}'_\perp|} = \sum_{l \geq 0} \sum_{m=-l}^l \frac{4\pi}{2l+1} \frac{q'^l_{\perp \leq}}{q'^{l+1}_{\perp >}} \tilde{Y}_{lm}(\frac{\pi}{2}, \phi') Y_{lm}(\frac{\pi}{2}, \phi), \tag{56}$$

where

$$P_l(\cos \gamma) = \frac{4\pi}{2l+1} \sum_{m=-l}^l \tilde{Y}_{lm}(\theta', \phi') Y_{lm}(\theta, \phi) \tag{57}$$

$$\begin{aligned}
P_0(x) &= 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1), P_3(x) = \frac{1}{2}(5x^3 - 3x), \\
\cos(\gamma) &= \sin(\theta) \sin(\theta') \cos(\phi - \phi') + \cos(\theta) \cos(\theta'),
\end{aligned}$$

for $\theta = \frac{\pi}{2}$ we get

$$\cos(\gamma) = \cos(\phi - \phi') \quad (58)$$

Thus the expansion of $\frac{1}{|q'_\perp - q_\perp|}$ becomes

$$\frac{1}{|q'_\perp - q_\perp|} = \frac{1}{q_\perp} \left[1 + \frac{q'_\perp}{q_\perp} \cos(\phi - \phi') + \left(\frac{q'_\perp}{q_\perp} \right)^2 \left(\frac{3 \cos^2(\phi - \phi')}{2} - \frac{1}{2} \right) + \dots \right] \quad (59)$$

Appendix C Form Factors Parametrizations

C.1 Sachs Electromagnetic Form Factors

The electric G_E and magnetic G_M form factors (known as Sachs form factors) are related to Dirac and Pauli form factors F_1 and F_2 as follows

$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned} \quad (60)$$

Here M is the nucleon mass. Writing F_1 and F_2 in terms of G_E and G_M we get ($\tau = \frac{Q^2}{4M^2}$)

$$\begin{aligned} F_1(Q^2) &= \frac{G_E(Q^2) + \tau G_M(Q^2)}{1 + \tau}, \\ F_2(Q^2) &= \frac{G_M(Q^2) - G_E(Q^2)}{1 + \tau}. \end{aligned} \quad (61)$$

For the electromagnetic form factors $G_E(Q^2)$ and $G_M(Q^2)$, the data fit from Ref. [19] were used.

C.2 Parametrization of Form Factors Using Generalized Parton Distributions

Generalized parton distribution (GPDs) can be considered as generalization of ordinary parton distributions. The formal definition of GPDs for transversely polarized nucleon but unpolarized quarks is given by

$$\langle p', S' | \hat{O}_q(x, \mathbf{b}_\perp) | p, S \rangle = \frac{1}{2P^+} \bar{u}(P', S') \left(\gamma^+ H_q(x, \xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M} E_q(x, \xi, t) \right) u(p, S) \quad (62)$$

Where $\bar{P}^\mu = \frac{1}{2}(P^\mu + P'^\mu)$ represents the average momentum of the target, $\Delta^\mu = P'^\mu - P^\mu$ is the four momentum transfer, $t = \Delta^2$ is the invariant momentum transfer and $\xi = -\frac{\Delta^+}{2P^+}$ is the change in the longitudinal component of the target momentum and is called the skewness. The nucleons form factors can be decomposed as follows

$$F_i^p = e_u F_i^u + e_d F_i^d + e_s F_i^s, \quad F_i^n = e_u F_i^d + e_d F_i^u + e_s F_i^s \quad (63)$$

Where $i = 1, 2$ and $e_u = \frac{2}{3}$, $e_d = e_s = \frac{-1}{3}$. The Dirac and Pauli flavor form factors at zero skewness are give by the following sum rules

$$F_1^q(t) = \int_{-1}^1 dx H^q(x, \xi, t), \quad F_2^q(t) = \int_{-1}^1 dx E^q(x, \xi, t) \quad (64)$$

The result of integration is independent of ξ . Also the integration region can be reduced to $0 < x < 1$ by introducing the non-forward parton densities

$$\mathcal{H}^q(x, t) = H^q(x, 0, t) + H^q(-x, 0, t), \quad \mathcal{E}^q(x, t) = E^q(x, 0, t) + E^q(-x, 0, t) \quad (65)$$

Where $q = u, d$ and $\mathcal{H}^q(x, t)$ reduces to the usual valence quark densities for $t \rightarrow 0$ for the up and down quarks. Now the form factors becomes

$$F_1^q(t) = \int_0^1 dx \mathcal{H}^q(x, t), \quad F_2^q(t) = \int_0^1 dx \mathcal{E}^q(x, t) \quad (66)$$

The magnetic densities satisfies the following normalization conditions

$$\begin{aligned} \kappa_q &= \int_0^1 dx \mathcal{E}^q(x), \quad \kappa_u = 2\kappa_p + \kappa_n = +1.673, \quad \kappa_d = \kappa_p + 2\kappa_n = -2.033 \\ F_2^p(t=0) &= 1.793, \quad F_2^n(t=0) = -1.913 \end{aligned} \quad (67)$$

Following [20], the anzats for the *GPDs*

$$\begin{aligned} \mathcal{H}^q(x, t) &= q_v(x) x^{-\alpha'(1-x)t}, \\ \mathcal{E}^q(x, t) &= \frac{\kappa_q}{N_q} (1-x)^{\eta_q} q_v x^{-\alpha'(1-x)t}, \end{aligned} \quad (68)$$

The normalization constants N_q satisfies

$$N_q = \int_0^1 dx (1-x)^{\eta_q} q_v(x) \quad (69)$$

Where the unpolarized parton distributions are parametrized as

$$\begin{aligned} u_v(x) &= 0.262x^{-0.69}(1-x)^{3.50}(1+3.83x^{0.5}+37.65x) \\ d_v(x) &= 0.061x^{-0.65}(1-x)^{4.03}(1+49.05x^{0.5}+8.65x) \end{aligned} \quad (70)$$

The parameters used in this fit are $\alpha' = 1.105, \eta_u = 1.713, \eta_d = 0.566$.

References

- [1] Andrei V. Afanasev, Stanley J. Brodsky, Carl E. Carlson, Yu-Chun Chen, and Marc Vanderhaeghen. Two-photon exchange contribution to elastic electron-nucleon scattering at large momentum transfer. *Phys. Rev. D*, 72:013008, Jul 2005.
- [2] S. Kondratyuk, P. G. Blunden, W. Melnitchouk, and J. A. Tjon. *Phys. Rev. Lett.*, 95:172503, Oct 2005.
- [3] A. Metz, D. Pitonyak, A. Schäfer, M. Schlegel, W. Vogelsang, and J. Zhou. *Phys. Rev. D*, 86:094039, Nov 2012.
- [4] Andreas Metz, Daniel Pitonyak, Andreas Schäfer, Marc Schlegel, Werner Vogelsang, and Jian Zhou. What causes transverse single-spin asymmetries in lepton-nucleon and in nucleon-nucleon scattering? In *International Journal of Modern Physics: Conference Series*, volume 25, page 1460011. World Scientific, 2014.
- [5] Y.-C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen. Partonic calculation of the two-photon exchange contribution to elastic electron-proton scattering at large momentum transfer. *Phys. Rev. Lett.*, 93:122301, Sep 2004.
- [6] Qattan et al. Precision rosenbluth measurement of the proton elastic form factors. *Phys. Rev. Lett.*, 94:142301, Apr 2005.
- [7] P. G. Blunden, W. Melnitchouk, and J. A. Tjon. *Phys. Rev. C*, 72:034612, Sep 2005.

- [8] Matthias Burkardt. *International Journal of Modern Physics A*, 18(02):173–207, 2003.
- [9] Gerald A. Miller. *Annual Review of Nuclear and Particle Science*, 60(1):1–25, 2010.
- [10] Carl E. Carlson and Marc Vanderhaeghen. *Phys. Rev. Lett.*, 100:032004, Jan 2008.
- [11] Daniel N. Kabat. Validity of the Eikonal approximation. *Comments Nucl. Part. Phys.*, 20(6):325–335, 1992.
- [12] Miguel S. Costa and Marko Djurić. *Phys. Rev. D*, 86:016009, Jul 2012.
- [13] J. Arrington, P.G. Blunden, and W. Melnitchouk. *Progress in Particle and Nuclear Physics*, 66(4):782 – 833, 2011.
- [14] M.E. Peskin and D.V. Schroeder. Advanced book classics. Addison-Wesley Publishing Company, 1995.
- [15] S. Collins et al. *Phys. Rev. D*, 84:074507, Oct 2011.
- [16] R. Mertig, M. Bohm, and Ansgar Denner. *Comput. Phys. Commun.*, 64:345–359, 1991.
- [17] Carl E. Carlson and Marc Vanderhaeghen. Two-photon physics in hadronic processes. *Annual Review of Nuclear and Particle Science*, 57(1):171–204, 2007.
- [18] Y.-W. Zhang et al. Measurement of the target-normal single-spin asymmetry in quasielastic scattering from the reaction ${}^3\text{He}^\uparrow(e, e')$. *Phys. Rev. Lett.*, 115:172502, Oct 2015.
- [19] W. M. Alberico, S. M. Bilenky, C. Giunti, and K. M. Graczyk. *Phys. Rev. C*, 79:065204, Jun 2009.
- [20] M. Guidal, M. V. Polyakov, A. V. Radyushkin, and M. Vanderhaeghen. *Phys. Rev. D*, 72:054013, Sep 2005.